Controlled interaction of co-rotating vortices

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Abstract

The two- and three-dimensional interactions of a pair of co-rotating vortices, representing a simplified model of flows found in the extended near wake behind aircraft wings, are analyzed using water tank experiments and numerical simulations. At low Reynolds numbers (Re), the vortices undergo two-dimensional merging, when the core size exceeds a certain fraction of the vortex separation distance. The time it takes to reach this limit increases with Re. At higher Re, a three-dimensional instability is observed, showing the characteristics of an elliptic instability of the vortex cores. The spatial structure of the amplified unstable modes, as well as their growth rate as function of axial wavelength are given and compared to theoretical predictions, showing excellent agreement between the three approaches. The instability is found to rapidly generate small-scale motion, initiating merging for smaller core sizes and producing a turbulent final vortex.

1. Introduction

For a number of years now, the problem of aircraft trailing wakes has been a matter of concern for air traffic management, airframe manufacturers and scientists throughout Europe. Many aspects of this complex issue have been identified and are currently studied in order to set optimal separation distances between civil transport aircraft, particularly in the vicinity of airports. Among these aspects, the safety and the characterization issues have been supported by the European commission through the funding of several projects within the Fifth Framework Programme (1998-2002). The present study takes partly place within the European project C-Wake: “Wake Vortex Characterization and Control”.

Characterization of wake vortices is prerequisite to the elaboration of any method of controlling them, i.e. solving the wake vortex problem. It consists in understanding the generic physical phenomena acting, from the generation of the wake down to the far field, where a vortex encounter with a following aircraft would take place. The present study focusses on the physics of the extended near wake, from just downstream of the aircraft wing generating a vortex sheet, to the region where the vortex system remaining in the far field (generally a single large vortex behind each wing) is established. In the region of interest, several concentrated vortex cores exist, formed by the roll-up of the vortex sheet, and some of them interact. One of the most common interactions observed between co-rotating vortices (vorticity of the
same sign) in near field of aircraft wakes is the merging of the vortices (see, e.g., [12,25]). Based on these observations, it is generally found that the wing tip vortex and the outboard edge flap vortex merge into a single vortex several wing spans behind the aircraft.

This process of like-signed vorticity collapse has been studied for a long time in a two-dimensional context [3,16,22,25,26], which was thought sufficient to describe the convective process in aircraft wakes. More recently, experimental studies in wind-tunnels [4,5,9] have clearly demonstrated the existence of small-scale unsteady phenomena during the convective vortex merging, apart from turbulent fluctuations and the ‘vortex wandering’ phenomenon. In a very recent experimental study, [19], it was shown that the temporal merging of initially symmetric co-rotating vortices can be three-dimensionally unstable. The unstable modes developing in the flow were identified as modes of the elliptic instability (see, e.g., [1,8,27,28]).

The aim of this study is to characterize this instability resulting in three-dimensional unstable merger, as opposed to the well-known two-dimensional stable merger. The flow considered here is a simplified model of systems of co-rotating vortices that can be observed in aircraft wakes, but which can be related to realistic aircraft flows (see section 5). The important points are that, on one hand, the physical mechanisms are identical to what may occur in realistic wakes and, on the other hand, that in the present configuration these mechanisms may be explored in great detail for characterization and control purposes. The model flows of both the experimental and the numerical study have been carefully generated and perturbed, in order to allow a rigorous control of the instability characteristics with respect to the base flow.

In addition, the combined experimental and numerical approach to the study of the 3D instability, including also elements from a theoretical analysis, represents a convincing mutual validation of the results, and allows exploration of certain details in a complementary way.

Section 2 describes the vortex pair flow under consideration, and gives details of the experimental and numerical methods. In section 3, the two-dimensional interaction, i.e. vortex merging, is briefly recalled, including a discussion of the effect of Reynolds number. Section 4 deals with the three-dimensional elliptic instability observed before merging at higher Reynolds numbers, showing its onset, spatial structure, wavelengths and growth rate, and the subsequent break-down of the flow. Conclusions, including a discussion of the relevance of the results for real aircraft wakes are given in section 5.

2. Technical details

2.1. The base flow

We consider a pair of viscous laminar vortices of equal circulation \( \Gamma \), which are initially straight, parallel, uniform along their axes, and without axial flow in their cores. In its basic state, the pair rotates, through mutual induction, around the midpoint between the two vortex centres, separated by a distance \( b \). Measurements have shown that the initial flow can be approximated remarkably well by a superposition of two Lamb-Oseen vortices, i.e. axisymmetric two-dimensional vortices with a Gaussian vorticity distribution

\[
\omega(r) = \frac{\Gamma}{\pi a^4} e^{-r^2/\alpha^2}, \tag{1}
\]

where \( r \) is the radial distance from the centre of each vortex, and \( \alpha \) the characteristic core radius. The latter is linked to the location \( r_{\text{max}} \) of maximum circumferential velocity by \( r_{\text{max}} \approx 1.12a \). Such flows are characterised by two non-dimensional parameters: the Reynolds number \( Re = \Gamma/\nu \), where \( \nu \) is the kinematic viscosity, and the non-dimensional core size \( a/b \). In a viscous flow, the core radius \( a \) grows in time by diffusion of vorticity according to:

\[
a^2 = 4\nu t + \text{const.} \tag{2}
\]

Time \( t \) is non-dimensionalised by the turnover period \( t_c = 2\pi b^2/\Gamma \) of a point vortex pair having the same circulation and (initial) separation distance as the pair under consideration, and the origin of time is chosen so that the constant in equation (2) vanishes.

2.2. Experimental set-up

In the experimental study, the flow was investigated in a rectangular water container with plexiglass walls, measuring \( 50 \times 50 \times 130 \text{ cm}^3 \). Vertical vortex pairs were generated in this tank using two long anodized aluminum plates, hinged on one side, and whose free edges were machined to an angle of 30°. They could be moved in a symmetric way by a computer-controlled step motor located outside the water and linked to the plates by a system of gears and belts. When the flaps are impulsively rotated, the boundary layers developing on each one separate, and roll up into two uniform and parallel starting vortices (see figure 1(a)). These vortices were typically separated by a distance of 3 cm, and the pair
Figure 1. 2D initial condition of the vortex pair flow. (a) Experiment at $Re = 2000$, $a/b \approx 0.15$, (b) DNS with $Re = 2500$, $a/b = 0.185$.

The flow was visualized using fluorescent dye, which was painted on the plates close to the sharp edges prior to their introduction into the water. Illumination was achieved with the light from a 5 W Argon laser, either in a plane perpendicular to the vortex axes, or in volume for side views of the three-dimensional structure. For quantitative velocity measurements, Particle Image Velocimetry (PIV) is used. The water is seeded with small reflecting particles and a digital camera captures pairs of images in sections perpendicular to the vortex axes.

The images are then treated with a cross-correlation algorithm (including window shifting and deformation) to extract the velocity and vorticity fields [20]. These measurements were used to determine the initial conditions of the flow after vortex formation. The Reynolds number varied in a range between 500 and 5000. The initial non-dimensional core radius $(a/b)_0$ was typically between 0.15 and 0.18.

Other measurements concerning the spatial structure and the growth rate of the instability were obtained from image analysis of flow visualizations recorded on video or numerically using a digital camera. More details on the experimental setup and methods can be found in [18,19].

2.3. Numerical method

Direct Numerical Simulations (DNS) and Large Eddy Simulations (LES) have been performed in the present study. Both are accomplished using the 3D compressible Navier-Stokes solver NSMB, developed partly at CERFACS, within a European consortium. This code uses a finite-volume discretisation of the Navier-Stokes equations. The temporal integration is achieved with a classical explicit four-stage Runge-Kutta scheme, which is fourth-order accurate when applied to a linear advection equation. The convective fluxes are discretised by a fourth-order skew-symmetric centred scheme developed at CERFACS, which was shown to have higher resolution capacities with respect to standard second-order schemes [7], especially in the presence of high spatial frequencies [12]. Viscous terms are discretised with a clas-
Figure 3. 2D merging at $Re = 1500$ visualised by contours of axial vorticity. $t^* = t/t_c$. Experiment: (a) $t^* = 1.4$, (b) $t^* = 1.8$, (c) $t^* = 2.3$; DNS: (d) $t^* = 1.3$, (e) $t^* = 1.9$, (f) $t^* = 3.1$.

The initial condition used in the simulations, consisting of a vortex pair with a given $(a/b)_o$, is obtained in the following way. Starting from a superposition of two circular co-rotating Lamb-Oseen vortices (equation (1)) with $(a/b)_1 < (a/b)_o$, a 2D direct numerical simulation is performed at a low Reynolds number ($Re = 2500$), until the core size has increased, by viscous diffusion, to the desired value. During this period, the vortices adapt non-linearly to each other on a non-viscous time scale, leading to a quasi-steady solution (in the frame of reference rotating with the vortex system) of the Euler equations, with elliptic streamlines near the vortex centres (see also [14]). An example of a two-dimensional adapted initial condition is given in figure 1(b). This 2D flow is then copied in the axial direction into each computational cross-flow plane, the axial velocity component being set to zero everywhere. In order to trigger the three-dimensional instability, a white random noise is added to all velocity components, whose amplitude is lower than $5 \times 10^{-3}$ times the maximum velocity of the base flow.

The axial dimension of the simulation domain is chosen sufficiently large to allow the natural selection and amplification of the unstable modes, and periodic boundary conditions are applied in this direction. The mesh is locally refined in cross flow planes in the region of the two vortices (see figure 2). Non-reflecting boundary conditions are applied at the lateral boundaries, which are located at least 6 times the initial vortex separation distance away from the vortices.
3. Two-dimensional merging

This section describes the two-dimensional merging process as observed in the present study. In the experiments, the flow remained two-dimensional for Reynolds numbers below approximately 2000. Higher Re could be explored via 2D calculations.

Starting from the initial condition given in figure 1, the 2D interaction happens in several different stages. In the first stage, the vortices rotate around each other like two point vortices, and the effect of each vortex on the other is weak. The separation distance $b$, shown in figure 4 for different $Re$, remains approximately constant, and the period of rotation is almost equal to the one for a point vortex pair ($t_c$). The oscillation of the separation distance around $b_0$ in the experimental measurements is due to the confinement of the flow by the vortex-generating plates. During this phase the core size $a$ increases according to equation (2).

The second stage begins when the vortices reach a critical size, scaled on the separation distance $b$. At this point, two tips of vorticity are created at the outer side of the vortices (figure 3(a,d)), which are subsequently ejected radially to form two arms of vorticity wrapping around the vortices (figure 3(b,e)). Meanwhile, the vortex centres get closer and rapidly merge into a single core. The separation distance decreases drastically within approximately half an initial rotation period, an interval which varies little with Reynolds number. This indicates that this second stage, i.e. the actual merging, is mainly a convective process, where diffusion of vorticity plays a minor role.

The agreement between experimental and numerical results, both for the time-dependent vorticity distributions and the evolution of the separation distance, is excellent. The “tail” in the latter, observed in the numerical result for values of $b$ lower than about 25% of $b_0$, is due to the persistence of two very low maxima in the central part of the newly formed vortex, which cannot be resolved in the experimental measurements.

The critical ratio of core size and separation distance, at which merging begins, is found to be $(a/b)_c = 0.25 \pm 0.01$ for the Gaussian vortices considered here. This result is close to the value of 0.32 above which merging of constant-vorticity patches is observed, as predicted by theoretical and numerical studies [22,26]. A more detailed assessment of the merging criterion for a number of different vorticity distributions, involving a suitably defined core size, can be found in [21].

In the third stage, the vorticity arms roll up around the central pattern (figure 3(b,e)), forming a spiral of vorticity, which is spread out and smoothed by diffusion (figure 3(c,f)), leading again to a practically axisymmetric final vortex. The core size of this final vortex then keeps increasing similarly to the Gaussian evolution (2), although the vortex is not exactly Gaussian after merging. When extrapolating the two core evolutions before and after merging into the merging time interval, one obtains an estimate for the increase of core size if the flow was inviscid. The present measurements show that the area of the final vortex is about twice the area of each initial vortex (see [19] for details).

The effect of Reynolds number is clearly seen in figure 4. It acts mainly on the first stage of the flow evolution, namely on its duration measured in convective time units (rotation periods). The time it takes for the core size to grow to the critical value, i.e. the time to the onset of merging, is proportional to $Re$, as confirmed by the measurements. In the spatially evolving flow behind an aircraft, this ‘time to merging’ would correspond to a ‘merging distance’ downstream. However, the result cannot simply be transposed, since other factors influence the ratio $a/b$ there, in particular the induction from other parts of the vortex sheet, which reduce the separation distance $b$ before merging. This leads to a much quicker increase of $a/b$ than the viscous growth of $a$ alone (see discussion in section 5).
4. Three-dimensional instability

When increasing the Reynolds number, the viscous phase before merging lasts sufficiently long for the development of a three-dimensional instability while the two vortices are still separated.

The side view visualizations in figure 5 show a clear observation of the three-dimensional instability. At this particular instant, the vortices, which spin around each other, are in a plane perpendicular to the view direction. The vortex centres are deformed sinusoidally with a wavelength $\lambda$ close to one vortex separation $b$, and the perturbations on both vortices are found to be in phase. This deformation mode, which is stationary in the rotating frame of reference of the pair, is very similar to what was recently observed in counter-rotating vortex pairs [15], where it was found to be a consequence of a cooperative elliptic instability of the vortex cores. Elliptic instability occurs in flows with locally elliptic streamlines [1,8,27,28], resulting here from the interaction between the vorticity of one vortex and the external strain induced by the opposite vortex. According to theory, the spatial structure and wavelength scale on the vortex core dimensions, i.e. on $a$. It is interesting to see that this instability still develops in a co-rotating configuration, whereas the long-wavelength Crow instability is suppressed by the rotation of the vortex pair [10].

The effect of the instability on the vortex pair is further illustrated in figure 6, where the distribution of axial vorticity is plotted. Under the influence of the instability, the total distribution in figure 6(a,c) is seen to lose its initial symmetry with respect to the midpoint between the vortices. In both vortices, the maximum of vorticity is displaced along a line at 45° to the one joining the two (to the right here), which corresponds to the stretching direction of the mutually induced strain, in agreement with elliptic instability theory. The symmetry properties of the flow allow to extract from this total flow the vorticity of the amplified instability mode, which is antisymmetric, as opposed to the base flow, which is symmetric with respect to the centre of the flow. The result in figure 6(b,d) reveals the characteristic two-lobe structure of positive and negative vorticity inside the vortex cores associated with the elliptic instability [15,28].

Further measurements were carried out to determine the growth rate $\sigma$ of the instability as function of axial wavelength. In the experiments, the wavelength could be selected by sticking small pieces of adhesive tape appropriately spaced along the vortex-generating plates. The growth rate was then determined from measurements of the time-dependent amplitude of the wavy centreline deformation obtained by flow visualisation. The flow field in the cross-section was obtained simultaneously using PIV, in order to precisely quantify the base flow conditions during the instability growth. More details about these difficult and time-consuming measurements are given in [13,19]. In the simulations, the axial extent of the computational domain was reduced to the one wavelength to be tested, thus reducing considerably the number of possible unstable modes. The growth rate at this wavelength was calculated from the growth of the amplitude of the associated Fourier mode [11,13].

The results are presented in figure 7 for two Reynolds numbers. At low $Re$ (figure 7(a)), a single, surprisingly large band of unstable wavelengths is found, ranging between 3 and 7 core radii approximately, and a most amplified wavelength near $\lambda = 4a$. The scatter in the experimental points is
due to the difficulty of making precise measurements in this very sensitive flow, and also to the fact that merging interferes very early with the development of the instability at these low Re. Nevertheless, the agreement with the numerical value, and also with the theoretical curve, is rather good. The latter was obtained by considering a single Gaussian vortex subject to an external strain rotating at the same frequency as the co-rotating pair (see [13] for details). It appears that the large bandwidth, which persists at much higher Re, is due to the particular vorticity distribution of the Lamb-Oseen vortex exposed to a rotating strain; it varies significantly with the ratio $a/b$.

At Reynolds numbers more representative of real aircraft wakes (figure 7(b)), other unstable bands appear in the numerical, as well as the theoretical results (these conditions are not accessible by our experiments). In the example shown, these bands overlap significantly, leading to a continuous interval of unstable wavelengths. As before, the qualitative and quantitative agreement between the different sets of results is very good.

When the perturbation amplitude gets sufficiently large, the organised spatial structure breaks down. At the locations where the vortex centres are most deformed, layers of fluid initially orbiting one vortex are drawn around the respectively other vortex in a periodic interlocking way. The corresponding tongues of dye are faintly visible in the experimental visualisation (figure 5(a)). This exchange of fluid (and vorticity) between the vortices, and the nonlinear interactions developing at large perturbation amplitude, have two consequences: 1. the vortices are drawn closer together, initiating a premature merging, and 2. the regular spatial structure of the flow breaks down almost explosively into small-scale motion during this merging, leading to a turbulent final vortex. In the low-Re case shown here, the flow eventually relaminarises, and one finds again a single viscous vortex at the end. Despite the 3D perturbation and intermittent turbulence, one can still define an effective core size. Measurements have shown that merging sets in much earlier, i.e. for lower $a/b$, than

**Figure 6.** Axial vorticity of the perturbed vortex pair before merging. Left: total flow. Right: vorticity perturbation only. Top: experiment at $Re = 3450$. Bottom: LES for $Re = 10^5$. 
Figure 7. Growth rate of the elliptic instability as function of axial wavelength. $\sigma^* = \sigma t_c$. ◊: experiment, •: DNS/LES, -: theory [13]. (a) $Re = 2700, a/b = 0.220$, (b) $Re = 10^5, a/b = 0.205$. 

in two-dimensional flow, and that the final vortex appears to be bigger than it would have been without the three-dimensional instability [19].

5. Conclusion and discussion

We have shown experimental and numerical results concerning the interaction of two co-rotating vortices in a model flow representing vortex patterns generated in the extended near wake behind an aircraft wing. In the two-dimensional case, the vortices undergo the well-known merging process, as soon as the time-dependent core size exceeds a critical value. However, the experimental study, where the flow is necessarily three-dimensional, has revealed the existence of a short-wave 3D instability, when the Reynolds numbers is above 2000. This instability is associated with a growing waviness of the vortex centre lines. The characteristic spatial structure of the instability mode and the comparison of growth rate measurements with theoretical predictions, have shown that this phenomenon is due to an elliptic instability of the vortex cores, which is a generic feature of strained vortical flows. It is here observed and analysed for the first time in a situation where the strain acting on a vortex is time-dependent, in this case rotating. The instability strongly influences the merging process, which sets in for smaller core sizes as in the 2D flow, and leads to small-scale motion and a turbulent final vortex. Qualitative and quantitative agreement between experimental and numerical results concerning these phenomena is found to be extremely good.
The present study focuses on low-Reynolds number flows in a rather simple configuration (laminar flow, symmetric vortices, no axial velocity), which has the advantage of allowing a precise identification and analysis of the physical phenomena acting. In real aircraft wakes, much more complex vortex systems generally develop, including multiple interacting counter- and co-rotating vortices. Nevertheless, the presence of two strong co-rotating vortices is a common feature observed behind aircraft wings in high-lift configuration, one being generated at the wing tip, and the other at the outboard flap edge. In many cases, these vortices experience merging due to the combination of two factors: 1. the roll-up of the vortex sheet generated all along the trailing edge of the wing, which increases the cores sizes \( a \) of the two vortices; 2. the action of the velocities induced by the other vortices that may have formed, or by the parts of the vortex sheet which have not yet rolled up, an effect which is generally observed to decrease the distance \( b \) between the principal co-rotating vortices. Thus both factors increase the rescaled core size \( a/b \), which is found to reach the critical value within a few wingspans, resulting in the merger of the vortices. The equivalent time to merging is thus found to be of the order of a convective time scale (rotation period of the pair), which is similar to what is observed in the present study, despite the enormous difference in Reynolds number \( (Re \sim 10^7) \) in aircraft wakes. Here, the increase of \( a/b \) is solely due to viscous diffusion of vorticity, which is fast at low \( Re \). As a result, the overall scenario is quite similar in both cases: a co-rotating vortex pair with a rapidly evolving \( a/b \) (covering a similar range of values), which undergoes merging within a few rotation periods. The results found here concerning the elliptic instability could therefore be relevant to real aircraft wake vortices.

The difference in Reynolds number has an effect on the growth rate of the instability. At values representing realistic situations, the flow may be considered as inviscid, which would lead to a higher growth rate than measured in the present study. The viscous correction terms are known, and predictions for high \( Re \) can easily be obtained [13]. Another consequence of a higher Reynolds number is the increased level of turbulence in and around the vortices. The characteristics of the turbulence spectrum in the region of the vortex cores may have an influence on the selection and the development of the unstable modes.

Other differences between model and ‘real’ flow exist: in an aircraft wake, the vortices generally have different circulations and core sizes. In addition, the strain experienced by each vortex, which is the origin of the 3D instability, is not uniquely due to the presence of the other co-rotating vortex, but may be a complicated function of the entire vortex system including the vorticity sheet. Both these effects have an influence on the wavelength of the instability, as well as its growth rate, but the basic phenomenon will still occur in most cases.

A major difference is the presence of axial flow in aircraft wake vortices, which may strongly affect the stability properties of the vortex system. Depending on the configurations, wake-like or jet-like axial velocity profiles are measured in the vicinity of one or both vortex cores. Under particular conditions, a helical instability is seen to develop in such flows [2,23,24]. A precise analysis of the effect of axial core velocities on the elliptic instability remains to be done.

Despite the differences that exist with realistic flows, the simplified model flow studied here nevertheless contains most of the important ingredients of co-rotating vortices generated behind an aircraft wing. The results presented in this paper, in particular concerning the three-dimensional elliptic instability, may therefore contribute to the understanding of the complex dynamics of aircraft wakes.

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References


