Study of a high-order Discontinuous Galerkin method / Finite Differences method coupled solver on hybrid meshes for CAA

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CONTEXT
Background

In the field of direct numerical simulation of acoustic waves in the presence of obstacles and/or an heterogeneous flow, both Discontinuous Galerkin (DG) methods and Finite Differences (FD) methods are widely spread. These methods hold specific advantages and drawbacks. In particular, we recall FDM [1] are rather easy to implement and show good diffusion and dispersion properties. On the other hand, they are not well adapted to take in account complex geometries, as they require to be run on structured meshes. Conversely, GDM [3] are very demanding in CPU and memory resources and are quite difficult to implement. On the other hand, they are well suited to take in account complex geometries as they might be run on unstructured meshes. Besides, their formulation allows local order refinement. Both these methods have been successfully implemented in dedicated solvers, over the years. Based on this, the idea of a heterogeneous method coupling in order to split the computational domain and/or locally take advantage of each method’s qualities has already been advanced in [2].

Motivations

The idea of such a coupling is then to be able to take into account complex boundary geometries and boundary conditions running a DGM on a fully unstructured mesh around the obstacles and a much cheaper FDM on a cartesian grid further away.

Moreover, a coupled solver approach allows a fully parallel design, so that DG and FD computations are run on different CPUs.

STRAtegy and PREliminary RESULTS

We approximate solutions of 2D Linearized Euler Equations (LEE) using DG and FD (4th order + 10th order method.

Implemented methods:

Finite differences method:
- Centered scheme of order up to 6 for gradients approximation.
- Linear multistep spatial high-frequencies filter contained 11 points (6th-order) stencil.
- Boundary conditions are imposed using ghost points. Note that the number of necessary ghost points is defined by the half-size of the largest centered stencil.
- Discontinuous Galerkin method (SPACE):
- Lagrangian basis of order up to 8 are implemented.
- We use fully upwinded numerical fluxes at elementary boundaries.

Time-integration method:
- In both solvers, we use a 3-step/3rd order Runge Kutta scheme.

(i) Compared performances of DGM and FDM, both on a cartesian mesh

Combing DGM (P2 basis) and FDM (4th order + 10th order filter) complemented by a common RK scheme leads to the following observations:

- DGM suffers a low optimal CFL (about 6 times lower than in the case of FDM).
- Control FDM schemes complemented by a high order filter offer a good control over numerical diffusion.
- In 2D cases, DGM requires about 2.6 times more single operations than FDM for a single gradient approximation.
- This leads to the the result presented in FIG. i-2.

(ii) Meshing strategy and coupling algorithm

a) Spatial approach

The coupling algorithm is based on a minimal overlapping of the FD grid and the DG computational mesh. Solvers do exchange informations at their boundaries. As a first approach, we impose two requirements on meshes’ relative position:
- All ghost points of the FD grid lie within the DG domain.
- All Gauss points of the DG computational boundary lie within the FD-domain.

It allows:
- To use high order interpolation of the solution field using the DG approach.
- To use Gauss points of the DG computational boundary to interpolate the solution field over the FD grid.

b) Temporal approach

For preliminary simplicity reasons, we impose both methods to use a common timestep and a common RK scheme. Solvers do exchange values at each RK sub-iteration.

Although the first attempts presented before show encouraging results, some further developments and analysis are necessary to ensure the efficiency of the method.

- High-order interpolation over the FD grid:
  In the presented examples, we only perform Q1 interpolation of the FD values in order to reconstruct the solution field at the DG domain’s boundary. This, of course, prevents from preserving the method’s spatial order. Interpolations of a higher order (both respects to the solver’s order) are therefore required (see FIG. iv-3).
- Numerical testing
  Next, a numerical study of the convergence rates of the error (in cases where an analytical solution is available) of the two-domain coupled solver is needed to make sure the high-order is globally maintained. Simultaneously, stability in the case of long-time computations has to be tested.
- Time-integration approach
  Improving common time-step and RK schemes in both solvers in a strong reduction. In particular, it may lead to a very low actual CFL in case of slightly heterogeneous cell size. This time can be saved by performing interpolations in time, and adapting the coupling algorithm.
- Applications and meshing-strategy
  In the forthcoming applications, mesh configurations in presented before will have to be adapted. The idea is to switch to a Chimera approach, with a non-matching (discontinuous) DG domain meant to contain an obstacle fully overlapped by a FD grid (case FIG. iv-3). This coupling algorithm between the two-domain case is then transferred to each coupling boundary, at the faces of the DG domain.

(iv) Future developments and analysis

Here, we present preliminary results in a first attempt to develop the study of a coupling algorithm between DG solver SPACE [4], and a dedicated FD solver. Our modelling is presently based on 2D Linearized Euler Equations (LEE) and rigid-wall boundary conditions. We also introduce elements of the foreseen developments, and the evaluation of the coupling method.